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CONTINUED FRACTION PATTERN OF DIRICHLET SERIES

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Abstract

In this paper we give a rational a correction function to the series. certainly improves the value of sum of the series and gives a approximation to it.Also correction function follows infinite continued fraction pattern.

1. Introduction

Commenting on the Lilavati rule for finding the value of circumference of a circle from its diameter, the commentator series for computing the circumference from the diameter. One such series attributed to illustrious mathematician Madhava 14^{th} century is

$$C = \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \cdots \pm \frac{4d}{2n-1} \mp \frac{4d\left(\frac{2n}{2}\right)}{(3n)^2+1},$$

where $+$ or $-$ indicates that n is odd or even and C is the circumference of a circle of diameter d.

Key Words : *Correction function, Error function,.*

2. TheDirichlet's Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$.

We know that Dirichlet series is convergent and converges to eta function.

Definition : If G_n denotes the correction function, then the error function is defined as

$$E_n = G_n + G_{n+1} - \frac{1}{(n+1)^2}.$$

Theorem : The correction functions for series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ follow an infinite.

Proof : We know that the correction function for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ is $G_n = \frac{1}{2n^2+2n+2}$. The corresponding function is

$$|E_n| = \frac{4}{\{(2n^2+2n+2)\}\{(2n^2+6n+6)\}(n+1)^2}.$$

The first order corection functionis $G_n[1] = \frac{1}{2n^2+2n+2}$. For further reducting error function, choose

$$G_n = \frac{1}{\{2n^2+2n+2\} + \frac{A_1}{\{2n^2+2n+2\}+x}}$$

where A_1 and x are any two real numbers. Then the error function $|E_n|$ is a minimum for $A_1 = 4$ and $x = 8$.

Again for reducing error, choose the correction function as

$$G_n = \frac{1}{\{2n^2+2n+2\} + \frac{4}{\{2n^2+2n+10\}+\frac{A_2}{\{2n^2+2n+1\}+x}}}.$$

Then $|E_n$ is a minimum when

$$G_n = \frac{1}{\{2n^2+2n+2\} + \frac{4}{\{2n^2+2n+10\}+\frac{9}{\{2n^2+2n+26}}}.$$

The third order correction function is

$$G_n[3] = \frac{1}{\{2n^2+2n+2\} + \frac{4}{\{2n^2+2n+10\}+\frac{9}{\{2n^2+2n+26}}}.$$

Continuing like this we get an infinite continued fraction

$$\frac{1}{\{2n^2+2n+2\} + \frac{4}{\{2n^2+2n+10\}+\frac{9}{\{2n^2+2n+26\}+\cdots}}}.$$

3. Application

The Dirichlet series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = {}^{n}(2) = \frac{\pi^2}{12}$.

We have ${}^{n}(2) = 0.8224670334$, using a calculator.

For $n = 10$, the series approximation after applying correction functions is given below.

Correction function	$S_n + (-1)^n G_n$
Without correction function	0.8179621756
$G_n[1]$	0.82246666801

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